

# Harmonic moments of the gluon density distribution in AA collisions

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By using Monte-Carlo implementations of  $k_T$ -factorization formula with running-coupling BK unintegrated gluon distributions for nucleus-nucleus collisions, we compute higher order harmonic moments of the initial density distribution for both RHIC(Au+Au@200GeV) and LHC(Pb+Pb@2.76TeV) collisions. We study their sensitivity to the size of the valence parton distribution in the nucleon.

## §1. Introduction

Spacetime evolution of the hot and dense system created from heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) or Large Hadron Collider (LHC) may be followed by hydrodynamical simulations. Indeed, the nearly perfect fluid picture appears to explain large elliptic flow discovered at RHIC and LHC. Recently, it was reported the effects of initial condition for hydrodynamics on the higher order flow harmonics in Au+Au collisions at RHIC<sup>1)</sup> by comparing various hydrodynamical models with different initial conditions.<sup>2)-4)</sup> Combined analysis of higher order flow harmonics may provide strong constraints on both the properties of QGP and the initial higher order moments of the density distribution of gluons in coordinate space.<sup>5)</sup> It was pointed out in Ref.<sup>6)</sup> that all moments have the same magnitude when there is no length scale for nucleons inside a nucleus, while their magnitude decreases with the harmonic number if a length scale is introduced. In this work, we compute higher order harmonic moments within Monte-Carlo version of  $k_t$  factorization formulation with running coupling BK equation, and study the effect of Gaussian width on the harmonic moments.

## §2. Theoretical Model

We will use the Monte-Carlo implementation of  $k_t$  formula with unintegrated gluon distribution function from numerical solutions of running coupling Balitsky-Kovchegov (MCrcBK)<sup>7),8)</sup> equation for the computation of gluon production in heavy ion collisions. This model is an extension of the Monte-Carlo KLN (MC-KLN) model.<sup>9)</sup> First, the nucleons in the two incident nuclei are randomly sampled according to the Woods-Saxon distribution which simulates the effects of fluctuations of position of hard color sources. At each grid point, we compute the thickness function  $T_A(\mathbf{r}_\perp)$  to obtain the local saturation scale and then compute the gluon density. Within a Gaussian nucleon approximation, thickness function (for the large- $x$  valence partons) is given by

$$T_p(r) = \frac{1}{2\pi B} \exp[-r^2/(2B)] . \quad (2.1)$$

Note that the mean square radius of the valence parton distribution corresponds to  $\langle r^2 \rangle = 2B$ . The probability of nucleon-nucleon collision  $P(b)$  at impact parameter  $b$  is

$$P(b) = 1 - \exp[-kT_{pp}(b)], \quad T_{pp}(b) = \int d^2s T_p(s) T_p(s-b). \quad (2.2)$$

where (perturbatively)  $k$  corresponds to the product of gluon-gluon cross section and gluon density squared. We fix  $k$  so that integral over impact parameter becomes the nucleon-nucleon inelastic cross section  $\sigma_{NN}$  at the given energy:

$$\sigma_{NN}(\sqrt{s}) = \int d^2b (1 - \exp[-k(\sqrt{s}) T_{pp}(b)]) , \quad (2.3)$$

with  $\sigma_{NN} = 61.36$  mb for  $\sqrt{s_{NN}} = 2.76$  TeV. Hence,  $P(b)$  broadens with increasing energy, even as the size  $B$  of the hard valence partons is fixed.

In this work, we vary the width of the Gaussian within the range  $B = 0.2, 0.3, 0.4$  fm<sup>2</sup>. Our model applies the  $k_t$ -factorized formula in the transverse plane perpendicular to the beam axis locally. The number distribution of produced gluons is given by

$$\frac{dN_g}{d^2r_\perp dy} \sim \frac{N_c}{N_c^2 - 1} \int \frac{d^2p_\perp}{p_\perp^2} \int^{p_\perp} d^2k_\perp \alpha_s \phi_A(x_1, (\mathbf{k}_\perp + \mathbf{p}_\perp)^2) \phi_B(x_2, (\mathbf{k}_\perp - \mathbf{p}_\perp)^2). \quad (2.4)$$

In MCrcBK,<sup>7)</sup>  $\phi$  is obtained from the Fourier transform of the numerical results of the running coupling BK (rcBK) evolution equation.<sup>10)</sup> Note that in (2.4) the small- $x$  evolution is *not* treated stochastically and that the density of produced gluons fluctuates only due to fluctuations of the large- $x$  valence charges.

### §3. Initial Harmonic moments

In Fig. 1, we plot the event average of eccentricities  $\varepsilon_n$  ( $n = 2, 3, 4, 5$ ) defined as<sup>11)</sup>

$$\varepsilon_n = \frac{\sqrt{\langle r^2 \cos(n\phi) \rangle^2 + \langle r^2 \sin(n\phi) \rangle^2}}{\langle r^2 \rangle} \quad (3.1)$$

where  $r^2 = x^2 + y^2$ ,  $x = r \cos \phi$ ,  $y = r \sin \phi$ .  $\langle \dots \rangle$  means the average in the transverse plane. Since the eccentricities may be proportional to the magnitude of flows, it is important to know the initial value of these quantities. The result from Monte-Carlo Glauber model in which eccentricity is calculated based on the positions of point-like participant nucleons that is labeled by Glauber (point-like) is shown in full squares. Monte-Carlo Glauber results assuming hard disc nucleon are plotted in open squares. In the hard disc approximation, the thickness function at each grid is obtained by counting the number of participant nucleons as  $T_A(\mathbf{r}_\perp) = \frac{\text{number of nucleon within } S}{S}$ , where the smearing area  $S = 42$  mb, at top RHIC energy. The finite size of the nucleons does not affect  $\varepsilon_2$  significantly, except for very peripheral collisions. However, Glauber with point-like nucleons yields larger values for higher moments ( $n \geq 3$ ) which is consistent with the results in Ref.<sup>6)</sup>

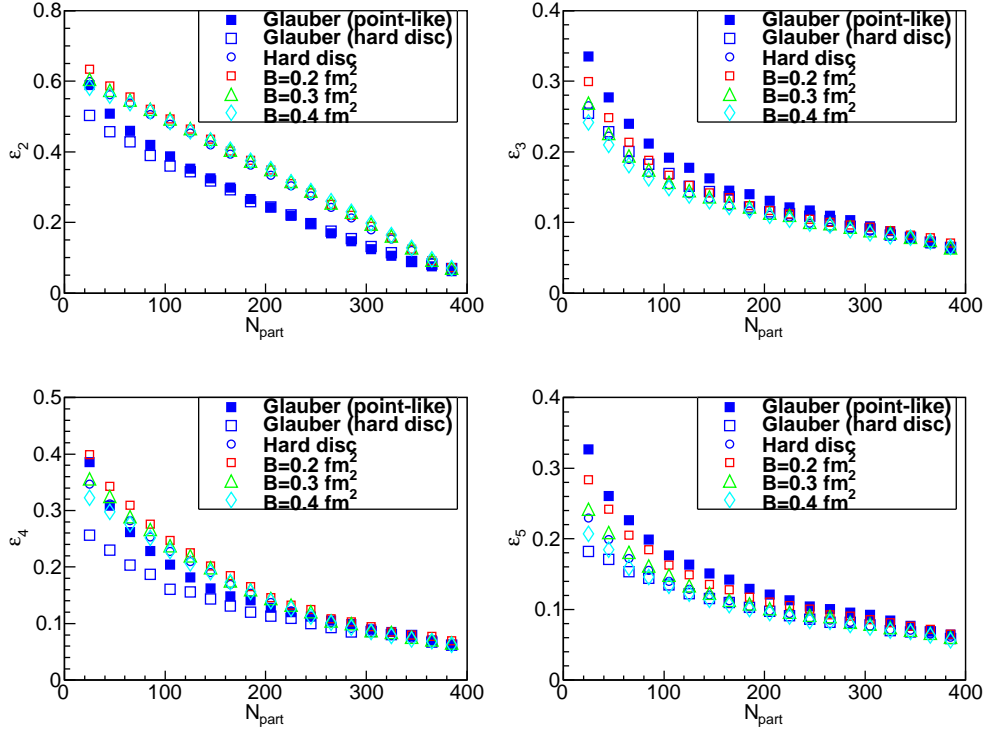


Fig. 1. Comparison of the centrality dependence of higher order harmonic moments for Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV.

On the other hand, one observes that the third and fifth harmonics are similar between the MC-Glauber (hard disc) and the MCrcBK models except for small value of Gaussian width  $B$ . in our model as pointed out in Refs.<sup>(12), (13)</sup>

We should mention that the prediction of  $\epsilon_3$  from a newly developed event generator DIPSY based on a dipole model is larger than that of the MC-KLN model,<sup>(14)</sup> while DIPSY prediction for  $\epsilon_2$  is the same as MC-KLN result. This may due to the additional fluctuations from BFKL cascade in DIPSY which is not included in our models in this work.

Figs. 2 show the harmonic moments for Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV and Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV from the MCrcBK model. One observes that as Gaussian width decreases, higher order harmonic moments increase. Therefore, it is important to check the sensitivity of higher flow harmonics to the length scale introduced by valence parton distribution in order to extract detailed information on the properties of quark-gluon plasma.

#### §4. Summary

We have presented results for higher order harmonic moments from the Monte Carlo version of  $k_t$  factorization formula with rcBK small- $x$  evolution (MCrcBK).

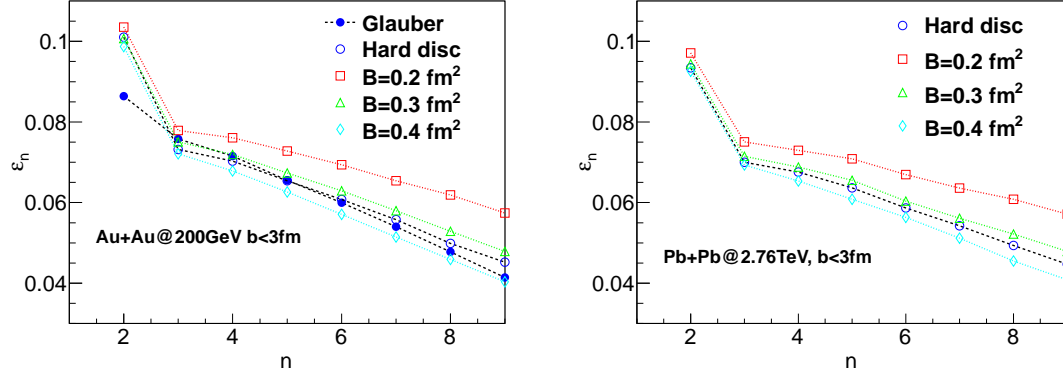


Fig. 2. Harmonic moments for central Au+Au collision ( $b < 3$  fm) at  $\sqrt{s_{NN}} = 200$  GeV (left) and Pb+Pb collision at  $\sqrt{s_{NN}} = 2.76$  TeV (right) from MCrcBK with various Gaussian width.

The present simulations account only for the fluctuations of the valence partons in the transverse plane. We have shown the length scale dependence of the harmonic moments of the initial density distribution. It will be interesting to see how this affects higher order hydrodynamical flows. Also, in the future it would be interesting to systematically explore the effect of additional fluctuations from the BFKL evolution ladders.<sup>14)</sup>

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### References

- 1) A. Adare *et al.* [PHENIX Collaboration], arXiv:1105.3928 [nucl-ex].
- 2) B. H. Alver, C. Gombeaud, M. Luzum and J. Y. Ollitrault, Phys. Rev. C **82**, 034913 (2010).
- 3) B. Schenke, S. Jeon and C. Gale, Phys. Rev. Lett. **106**, 042301 (2011).
- 4) H. Petersen, G. Y. Qin, S. A. Bass and B. Muller, Phys. Rev. C **82**, 041901 (2010).
- 5) R. A. Lacey, R. Wei, N. N. Ajitanand and A. Taranenko, Phys. Rev. C **83**, 044902 (2011).  
R. A. Lacey, A. Taranenko, N. N. Ajitanand and J. M. Alexander, arXiv:1105.3782 [nucl-ex].
- 6) P. Sorensen, B. Bolliet, A. Mocsy, Y. Pandit, N. Pruthi, [arXiv:1102.1403 [nucl-th]].
- 7) J. L. Albacete and A. Dumitru, 1011.5161[hep-ph],  
[http://physics.baruch.cuny.edu/node/people/adumitru/res\\_cgc](http://physics.baruch.cuny.edu/node/people/adumitru/res_cgc)
- 8) J. L. Albacete, A. Dumitru, Y. Nara, J. Phys. Conf. Ser. **316**, 012011 (2011).
- 9) H. J. Drescher and Y. Nara, Phys. Rev. C **75**, 034905 (2007); **76**, 041903(R) (2007).
- 10) J. L. Albacete and Y. V. Kovchegov, Phys. Rev. **D75**, 125021 (2007).
- 11) B. Alver, G. Roland, Phys. Rev. **C81**, 054905 (2010).
- 12) G. -Y. Qin, H. Petersen, S. A. Bass, B. Muller, Phys. Rev. **C82**, 064903 (2010).
- 13) Z. Qiu, U. W. Heinz, Phys. Rev. **C84**, 024911 (2011).
- 14) C. Flensburg talk in this conference, [arXiv:1108.4862 [nucl-th]].